GLOBAL IMPACT OF U.S. INTEREST RATE

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Abstract. There have been many empirical analysis on the financial power of the Fed, with its weapon, that is, the monetary policy. Most of them agree that the global economy will be drastically affected by even slight changes in the monetary policy decided by the Fed. However, most of them are done in qualitative analysis in the field of Economics, with few in quantitative analysis. In this paper, we use a commonly used analysis tool in Econometrics to statistically analyze the global impact of changes in U.S. interest rate on the global economy, and ultimately interpret the result of the model.

1. Introduction

Since the 1980s, neoliberalism has gradually become the mainstream philosophy of Western economics, while serving as the economic guideline to consolidate the world as a whole. As a result, economists have been studying the ripple effect of a change in U.S. interest rate on the entire world. In this paper, we introduce a mathematical method to analyze this effect.

1.1. **Interest Rate.** The interest rate of a country or nation, refers to the target interest rate set by its central bank, to achieve specific economic goals. For the United States, it is called the federal funds rate, which is an interval instead of a point. However, a point estimate can be calculated using the weighted median of the overnight federal funds rate charged by depository institutions to each other, which is called the Effective Federal Funds rate.

1.2. **Real GDP per Capita.** A recession is a significant, widespread, and prolonged downturn in economic activity. To measure recessions, one simple economic indicator is the real Gross Domestic Product (GDP) per capita, which is dividing annual real GDP over mid year population. It excludes the underlying effects of inflation and population growth from the GDP.

The changes in the federal funds rate is part of the monetary policies which are intended to increase or decrease money supply in the market. However, this effect does not only affect the domestic market, but also the international market with the "spill over" effect[1]. To mathematically measure this impact on the world economy, we will use a popular tool called Vector Autoregression, developed by Sims (1980)[2]. It is a simple but powerful model developed as an alternative to the traditional complex models. We will use this model to measure the impact of increases and decreases in the United States the Effective Federal Funds rate since 1960 on global real GDP per capita.

2. Time Series

The two economic indicators mentioned above, the Effective Federal Funds rate and real GDP per capita, are time series.

Definition 2.1 (Time Series)**.** A time series is a series of data points indexed (or listed or graphed) in time order. Usually denoted as y_t or x_t.

A time series is similar to a stochastic process, which is usually defined as a sequence of random variables, where the index of sequence have the interpretation of time. However, compared to the stochastic process, one of the most important characteristics of a time series is stationarity. Many analytical tools and statistical tests and models rely on the assumption that the time series are stationary[3]. In order to use the analysis tool in the following sections, we need to have stationary time series.

Definition 2.2 (Stationary)**.** Let $\{X_t\}$ be a stochastic process and let $F_X(x_{t_1+\tau}, \ldots, x_{t_n+\tau})$ represent the cumulative distribution function of the unconditional joint distribution of $\{X_t\}$ at times $t_1 + \tau, \ldots, t_n + \tau$. Then $\{X_t\}$ is strictly/strongly stationary if:

$$F_X(x_{t_1+\tau}, \ldots, x_{t_n+\tau}) = F_X(x_{t_1}, \ldots, x_{t_n}).$$

Loosely speaking, a process is strongly stationary if its statistical properties do not change over time, or depend on time. The definition of strong stationarity implies that its mean, variance and covariance stay constant over a selected time interval for different time periods. However, in real life situations, most of the economic time series will not satisfy this strict condition, since they will be strongly affected by many external factors. If we wish to use tools that rely on stationarity, we need to introduce a weaker definition of stationarity, namely trend stationarity.

Definition 2.3 (Trend Stationary)**.** A trend stationary process, is a stochastic process from which an underlying trend can be removed, leaving a stationary process.

In other words, if we could identify the trend in the time series and then remove it, it would result in a stationary time series, which we could then analyze. The most common type of trends are linear, which could be modeled as $bt + a$, polynomial, which could be modeled as $bt^n + ct^{n-1} + \ldots + a$ and cyclical or seasonal trends, which could be modeled using periodic functions.

Since stationarity implies a stable mean, variance and covariance, if we find ways to stabilize those factors to an acceptable range, the adjusted time series may become stationary. One method to stabilize the mean is differencing[4], which is to take the differences between consecutive values of the time series. This procedure may be repeated multiple times until resulting in a stationary time series.

Definition 2.4 (Order of Integration)**.** Let y_t be a time series, and let $n \in \mathbb{N}$. Then the nth difference of y_t is,

$$\nabla^n y_t = \nabla^{n-1} y_t - \nabla^{n-1} y_{t-1}.$$

The minimum number of differences required to obtain a stationary time series, is called order of integration, denoted as $I(d)$. Therefore, we could keep differencing a time series until its mean becomes stable and the time series itself becomes stationary.

Similarly, one method to stabilize the variance is the logarithm. Either natural log or log base 10 may be used. This is extremely useful if the time series is believed to exhibit exponential growth, or if its magnitude is deemed to be extremely large.

Example 2.5. In classical economic theory, the GDP of an perfect ideal country is believed to grow exponentially, which makes it non-stationary. Therefore taking logarithm of the original data, and then differencing it as needed, it will result in a stationary time series, to which analysis tools could be applied.

3. Vector Autoregression

Definition 3.1 (Autoregression)**.** Let y_t be a stationary time series. Then the Autoregression (AR) model which assumes y_t is dependent on its own previous values, and on a stochastic term e_t, has the following general form,

$$y_t = c + a_1 y_{t-1} + a_2 y_{t-2} + \ldots + a_p y_{t-p} + e_t,$$

where the a_i are the coefficients for previous values of y_t, c is the intercept, the lag order p indicates how many terms are included, and e_t is the error term (or residual), which ideally follows a normal distribution with mean of 0 and some variance σ^2.

Definition 3.2 (Vector Autoregression)**.** Let $y_{k,t}$ be a series of k stationary time series.

$$y_{1,t} = c_1 + a_{1,1}y_{1,t-1} + \ldots + a_{1,p}y_{1,t-p} + \ldots + a_{k,1}y_{k,t-1} + \ldots + a_{k,p}y_{k,t-p} + e_{1,t}$$
$$\vdots$$
$$y_{k,t} = c_k + a_{1,1}y_{1,t-1} + \ldots + a_{1,p}y_{1,t-p} + \ldots + a_{k,1}y_{k,t-1} + \ldots + a_{k,p}y_{k,t-p} + e_{k,t}$$

Note that external variables and deterministic trends or seasonality could be easily added into this model. For example, we could add b_1t as a linear function of time to indicate an increasing trend, into the first equation in the above model. Note also that the above system of equations could be expressed in matrix form.

Definition 3.3 (VAR in Matrix Form)**.** Let Y_t be a column vector of k stationary time series.

$$Y_t = C + \sum_{i=1}^{p} A_i Y_{t-i} + \epsilon_t$$

where ϵ_t is a $k \times 1$ matrix or column vector, containing residuals. Each A_i is a $k \times k$ square matrix containing coefficients, and C contains the intercepts.

The coefficients are estimated for a given series of data using Ordinary Least Squares (OLS) equation by equation, which is a method for choosing the unknown parameters in a linear equation. However, for vector autoregressions, it is more important to examine the residuals instead of the coefficients. We have mentioned that each e_t should follow a normal distribution, so the residuals ϵ_t as a collection of e_t, should follow an independent and identical distribution (IID, if each random variable has the same probability distribution as the others and all are mutually independent.) with a mean of 0 and a variance of Σ, which we will define below.

Definition 3.4. The matrix Σ is a $k \times k$ positive definite symmetric square matrix containing the covariances.

$$\Sigma = \begin{bmatrix} \sigma^2_{y_1} & \sigma_{y_1y_2} & \cdots & \sigma_{y_1y_k} \\ \sigma_{y_2y_1} & \sigma^2_{y_2} & \cdots & \sigma_{y_2y_k} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{y_ky_1} & \sigma_{y_ky_2} & \cdots & \sigma^2_{y_k} \end{bmatrix}$$

Since the covariance matrix is symmetric, the covariance of two variables only indicates how strongly they are related, but it does not convey causal relationship from one variable to another variable. For instance, if changes in x_t causes same-direction changes in y_t, which also causes changes in z_t, only looking at the covariances and correlations will not reveal the real causations among those three variables. Another limitation is that each variable is only dependent on the past values, but not present values of other variables. If a variable reacts to a change in another variable in a short enough period, this reaction can not be properly captured by this model.

In fact, the above model is the reduced form vector autoregression, which has limitations as described above. Therefore, we need to introduce a stronger form of vector autoregression.

Definition 3.5 (SVAR)**.** Let Y_t be the time series in vector form. Ignoring the trend, the structural vector autoregression (SVAR) has the following general form,

$$A_0Y_t = C + A_1Y_{t-1} + A_2Y_{t-2} + \ldots + A_pY_{t-p} + \epsilon_t,$$

where the main diagonal terms of A_0 are scaled to 1, and all elements in the off diagonal of the covariance matrix Σ are 0.

However, using OLS methods would yield an infinite set of different values for each A_t, which all imply exactly the same probability distribution for the observed data[5]. Therefore, we need to introduce restriction matrices so that the coefficient estimators are certain.

Definition 3.6. Let Y_t be the time series in vector form. Ignoring the trend, the vector autoregression with a restriction matrix A has the following general form,

$$AY_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + \epsilon_t$$

Without loss of generality, A is limited to be a lower triangular matrix, where the diagonal is scaled to 1, and other non zero off diagonal entries are intentionally chosen. In this way, the coefficients could be estimated using OLS just like the reduced form. Here is the general form of A,

$$A = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ a_{21} & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & \ldots & 1 \end{bmatrix}$$

Similarly, we could also place a restriction matrix right before the residual, which plays a similar role as the restriction matrix before the variable itself.

When regressing with one variable, the reason to choose a lower triangular matrix instead of an upper triangular matrix is that when multiplying the matrix, the variables vector could be ordered such that the variables that respond to this specific variable contemporaneously are ordered first, followed by the variable itself, and then all other variables that do not respond contemporaneously. In addition, the entries in the restriction matrices are chosen based on economic theory or prior knowledge, which could be different for different researchers.

However, a critique against the reduced form VAR models is the problem of overparameterization, since it is often useful to deal with huge amount of data. This could be improved by using Bayesian VAR which treats coefficients as random variables instead of constants.

4. Tests

The fundamental requirement to construct a VAR model is that the time series are stationary, or at least trend stationary. To test for stationarity, the tool used is the unit root test, which tests whether the time series possesses a unit root. Several unit root tests have been developed, and one of most commonly used is the Augmented Dickey Fuller (ADF[6]) test.

In order to derive the formula for the ADF test, let's rewrite the general form of vector autoregression in terms of differences.

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \ldots + a_p y_{t-p} + e_t$$

$$y_t - y_{t-1} = (a_1 - 1) y_{t-1} + (a_2 + a_3) y_{t-2} + a_3 (y_{t-3} - y_{t-2}) + \ldots + e_t$$

$$y_t - y_{t-1} = (a_1 + a_2 + a_3 - 1) y_{t-1} + (a_2 + a_3)(y_{t-2} - y_{t-1}) + a_3 (y_{t-3} - y_{t-2}) + \ldots + e_t$$

$$\vdots$$

$$y_t - y_{t-1} = (a_1 + a_2 + a_3 + \ldots - 1) y_{t-1} + (a_2 + a_3 + \ldots)(y_{t-2} - y_{t-1}) + (a_3 + \ldots)(y_{t-3} - y_{t-2}) + \ldots + e_t$$

Let $\Delta y_t = y_t - y_{t-1}$ for all t, and $\gamma = a_1 + a_2 + a_3 + \ldots - 1$, then

$$\Delta y_t = \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + \ldots + e_t$$

Definition 4.1 (ADF Test)**.** Let y_t be a time series. With a linear function of time, the Augmented Dickey Fuller test is testing if $\gamma = 0$ in the following equation.

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \delta_2 \Delta y_{t-2} + \ldots$$

$$t = \frac{\hat{\gamma}}{SE(\hat{\gamma})} + e_t$$

where $\hat{\gamma}$ is the least squares estimate of γ and $SE(\hat{\gamma})$ is the usual standard error estimate.

The null hypothesis is that the time series is not stationary, and the alternative hypothesis is that the time series is stationary. The intuition is that if $\gamma = 0$, the time series ends up being a random walk, which is always non stationary. Note that, the ADF test has its own table of critical values.

The next test is the Granger Causality test, which tests whether the past values of one variable contain information that helps predict changes in another variable.

Definition 4.2 (Granger Causality)**.** Let y_t and x_t be time series with selected lag order p. Then the equation to test whether x_t is Granger causing y_t is,

$$y_t = \sum_{i=1}^{p} A_1 y_{t-i} + \sum_{i=1}^{p} A_2 x_{t-i} + \epsilon_t$$

The null hypothesis is that the coefficients of x_t are all zero, and the alternative hypothesis is that at least one of the coefficients of x_t is significantly different from zero. In fact, x_t could be treated as a collection of exogenous variables together (which are independent variables or outside factors in the model), instead of one single variable. However, as the name suggested, Granger causality is not necessarily true causality. One limitation for it is that it only identifies the cause-effect relations with constant conjunctions. If y_t and x_t are both driven by an exogenous variable, which is not included in the system, one may not be able to reveal the true causation relationships.

Note that in both tests and the VAR model itself, the lag order p needs to be selected carefully, which could be done with the multiple selection criteria[7], defined below.

Definition 4.3. Some common lag order selection criteria are defined below[8].

$$AIC(p) = \ln|\Sigma\bar{(}p)| + \frac{2}{N}(K^2 p)$$

$$FPE(p) = \left(\frac{N + p*}{N - p*}\right)^k |\Sigma\bar{(}p)|$$

where N is the sample size, k is the number of variables, $p*$ is the total number of parameters in each equation and $\bar{\Sigma}$ is the quasi maximum likelihood estimate of the covariance matrix. The lag order p is chosen to minimize the values of those criterion functions.

Note that choosing different lag orders could tremendously alter the test results. If p is too small, the remaining serial correlation in the errors will bias the test result. If p is too large, then the power of the test will suffer as a result of too few degrees of freedom.

5. Data

Now we are able to conduct the vector autoregression in R, with five annual time series from 1971 to 2020: the Effective Federal Funds rate as percent change from last year (EFF), the Consumer Price Index for United States (CPI), the GDP growth rate for United States (USA), the GDP growth rate for Developed countries (DEV) represented by High Income countries[1], and the GDP growth rate for Emerging markets (EME) represented by Middle Upper Income countries[2]. For data availability reason, we are dealing only with annual data, therefore, we are going to set the alpha level to 10%. Figure 1 shows the graph for three of the five selected time series since 1971 to 2020[3].

First, we conduct the ADF test to test the stationarity of those time series. The first four time series all have the resulting p-values less than 0.05, and the last time series has a p-value greater than 0.05 but less than 0.10. Therefore, we conclude that all time series are at least trend stationary.

[1]Partial list of High Income countries: Australia, Austria, Canada, France, Germany, Israel, Japan, Korea Rep., Luxembourg, Singapore, United Kingdom, United States.

[2]Partial list of Middle Upper Income countries: Argentina, Brazil, China, Cuba, Iraq, Malaysia, Mexico, Peru, Russian Federation, South Africa, Thailand.

[3]Data source: World Bank, Federal Reserve Economic Data, Organisation for Economic Co-operation and Development.

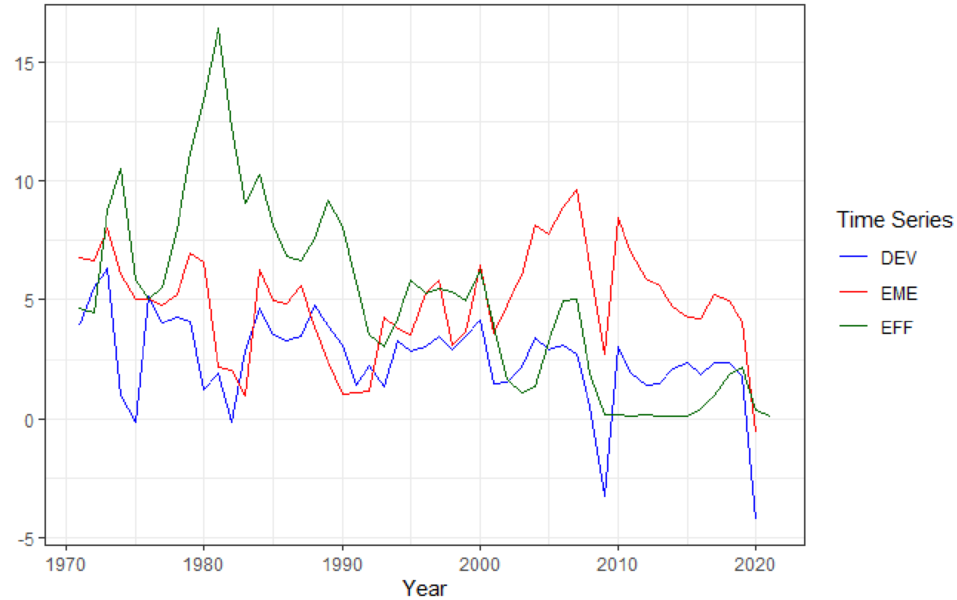

Figure 1. Selected three annual time series from 1971 to 2020

Now, we select our lag order, which is going to be 1 based on the two selection criterion defined above. Table 1 shows results for the values of those selection criterion with the lag order up to 5.

	1	2	3	4	5
AIC(p)	1.5350	1.7152	1.7392	2.1250	1.8664
FPE(p)	4.6659	5.7347	6.2779	10.5409	10.2999

Table 1. Selected selection criterion for lag orders up to 5.

Then we conduct the Granger causality test from one time series to another, with selected lag order equal to 1. The test results are quite interesting as they show that both the Effective Federal Funds rate and the Consumer Price Index are Granger causing both the GDP growth rate for United States and for the Developed market countries, but not significantly for the Emerging market countries. This is consistent with an assumption that those Developed countries are closely following the monetary and fiscal policies of United States, thus producing a strong correlations with the economy of United States[9].

With all the preliminary work done, we are able to estimate the coefficients for the vector autoregression equations, each with a constant. Here are the two most important equations which we will be analyzing,

$$\mathrm{DEV} = 0.0114\mathrm{EFF}_{t-1} - 0.0219\mathrm{CPI}_{t-1} + 0.3402\mathrm{USA}_{t-1} + 0.0262\mathrm{DEV}_{t-1} - 0.1061\mathrm{EME}_{t-1} + 2.0251$$

$$\mathrm{EME} = -0.0743\mathrm{EFF}_{t-1} - 0.0124\mathrm{CPI}_{t-1} + 0.6534\mathrm{USA}_{t-1} - 0.7196\mathrm{DEV}_{t-1} + 0.6151\mathrm{EME}_{t-1} + 2.2868$$

6. Results

6.1. **Residuals.** Residuals are the difference between observed data and predicted data using estimated equations, or the errors that could not be explained by regressing with only endogenous variables, which are variables that are changed or determined by their relationship with other variables within the model.

Essentially, it is the unexpectedness from regressing only endogenous variables. So a large residual in absolute value means there is a shock from exogenous variables which is not properly captured. We see this in our vector autoregression model above since in real life, there are more variables that could affect

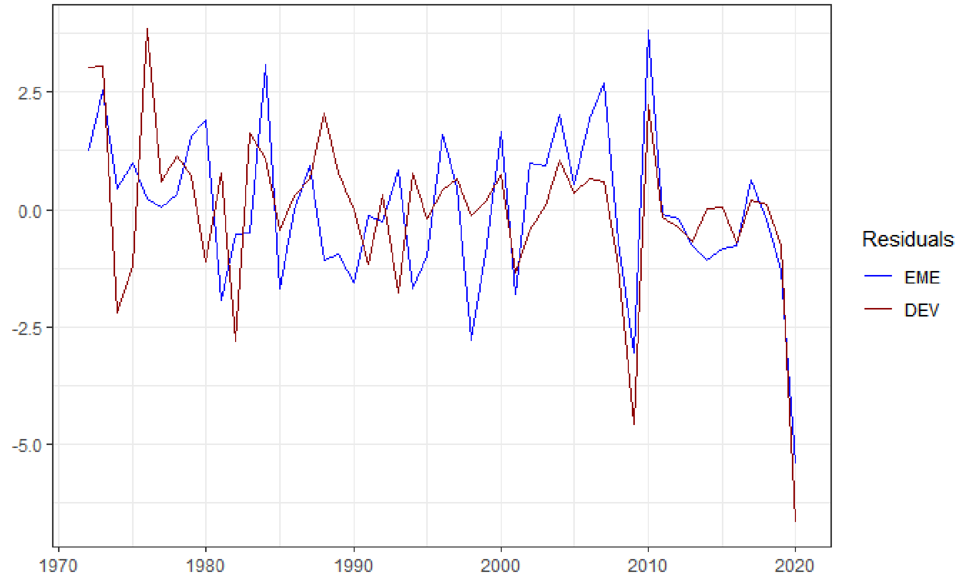

Figure 2. Residuals in GDP Growth Rate for DEV and EME

economic growth, and there are also global financial crises that no one would expect. Note that in Figure 2, the greatest declines in global economy happened around 1982, 2009, and 2020 which all result from the global recessions in previous year due to using a lag order of 1. The greatest single decline, due to the 2019 global pandemic, was much greater than the declines in 2008 and 2009, which were due to the subprime mortgage crisis followed by a global financial crisis.

6.2. **Impulse Response Function.** The impulse response function (IRF[10]) measures the changes of one variable in reaction to a one-standard-deviation shock from another variable. The IRF function shows how

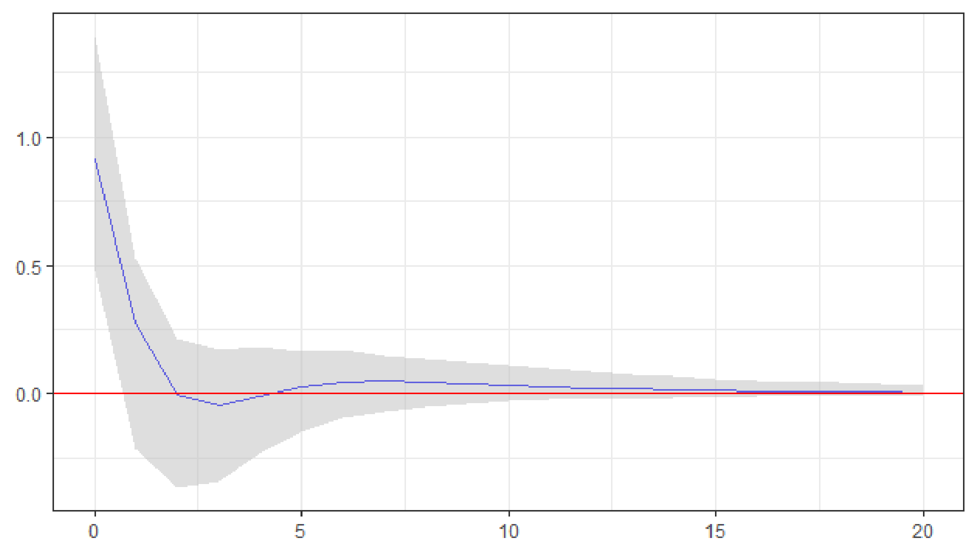

Figure 3. Orthogonal Response from EFF to DEV, 95% CI, 100 Runs

a shock $\epsilon_t = \delta$ in one variable at given time, impacts another variable looking n period ahead, assuming no further shocks. In this way, we could filter out the interference of other endogenous variables and focus on the shock transmitted only from one variable to another. Figures 3 and 4 illustrate this function for two pairs of variables with the shade representing 95% confidence intervals and the blue line represents the impulse response function value.

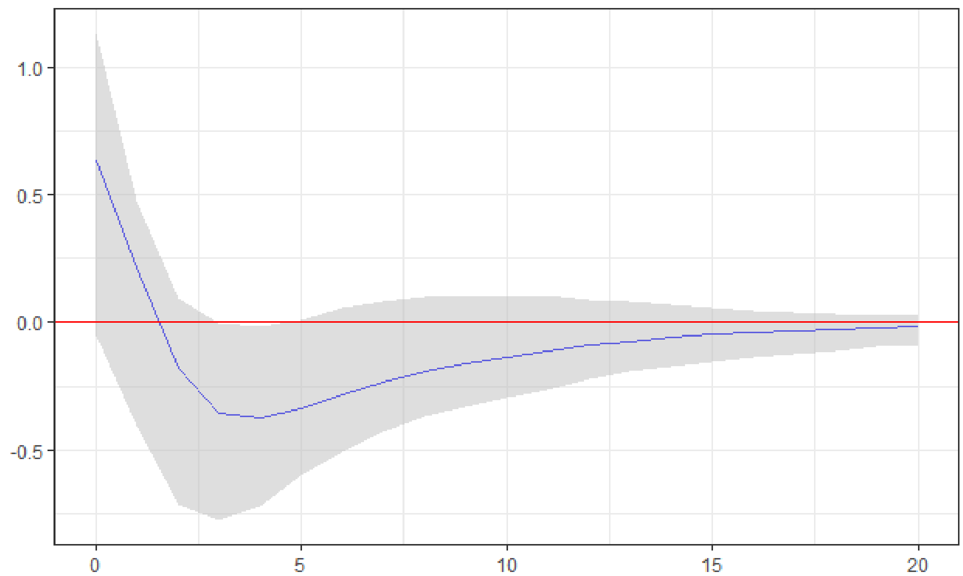

FIGURE 4. Orthogonal Response from EFF to EME, 95% CI, 100 Runs

The results show consistency with the current economic theory about the monetary shocks[11], as once the Fed raises its interest rate, the majority of nations would experience decrease in their GDP, with a lag. However, different countries will experience different level of impact, due to different policies across countries. For Developed countries like High Income countries (Figure 3), an increase in the Effective Federal Funds rate will decrease the GDP growth rate to zero, then negative, but back to positive after 4.5 years, and eventually diminishing to 0. In contrast, for Emerging markets like Middle-Upper Income countries (Figure 4), the impact is considerably greater, and it is likely to persist for multiple periods until it converges to zero.

6.3. **Variance Decomposition.** A forecast error variance decomposition (FEVD[12]) indicates how much of the forecast error variance of each of the variables could be explained by exogenous shocks to the other variables. It is useful in determining the importance of a variable in explaining another variable in the long term. For instance, in the system of aggregate supply and demand, the changes in aggregate demand will only affect short-run aggregate supply, but not long-run aggregate supply. The best way to examine the variance decomposition is to visualize them, and Figures 5 and 6 below are the results for selected time series looking 10 periods ahead.

For the variance decomposition of DEV, over half of the error comes from the GDP growth rate for United States, and the second largest factor comes from EFF. This means that actions in the United States are the major contributing factors in causing unexpected changes in predicting economies of Developed countries. In constrast, for the variance decomposition of EME, the largest sector comes from EME itself, meaning the economies of Emerging countries themselves are causing the largest portion of unexpectedness in forecasts, which is a sign that economies in Emerging market countries are less stable than those in Developed countries.

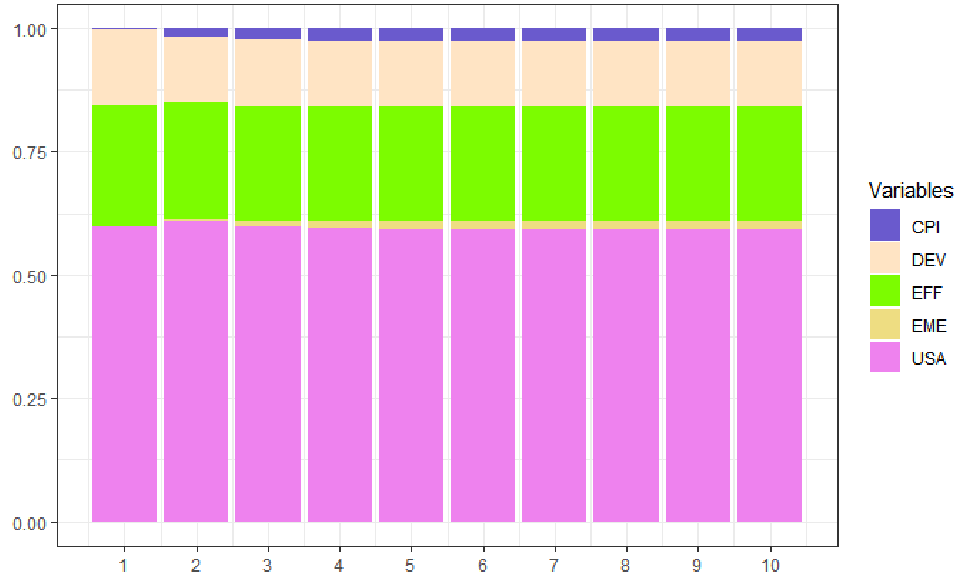

FIGURE 5. Forecast Error Variance Decomposition for DEV with 10 periods

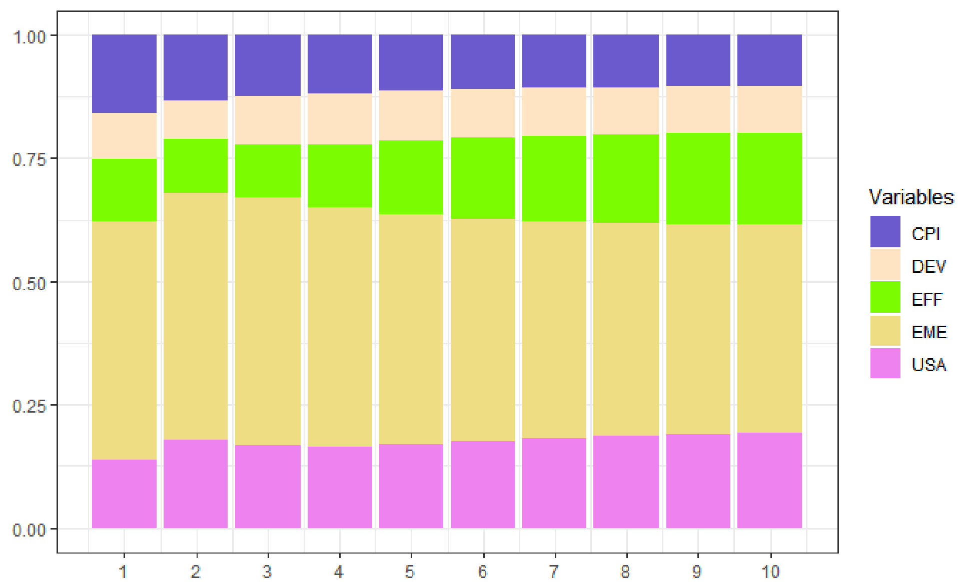

FIGURE 6. Forecast Error Variance Decomposition for EME with 10 periods

From the results of both IRF and FEVD functions, and the VAR equations itself, we conclude that the increases in the Effective Federal Funds rate would cause a decline in the GDP grow rate for most countries, although the severity and length of this impact would vary from country to country. Emerging markets are likely to experience a more severe and longer-lasting impact as it will last for couple of years until it diminishes, whereas Developed countries would experience a less severe impact, and are able to recover more quickly. One important insight is that for some countries, it may even be an opportunity for them, since they could attract more capital fleeing from those Emerging markets countries being affected.

7. Conclusions

The Vector Autoregression model above achieves our goal pretty well, although it has some limitations. From one perspective, the greatest advantage of the Vector Autoregression is that the only prior knowledge required is the values of variables which could be hypothesized to have causal relationship to each other. The calculation is fairly simple as coefficients in each equation can be estimated using Ordinary Least Squares, which surpasses the earlier complex modeling. However, as noted, it does not properly capture the contemporaneousness within the same period. For instance, the Effective Federal Funds rate changes from day to day, but each country measures its GDP only on a quarter-to-quarter basis. Moreover, there is a time lag between the end of each quarter and the publication of GDP data. In addition, the Vector Autoregression itself is not supported by any economic theory since it is purely a mathematical model with predetermined assumptions.

If we have the proper knowledge of economic theories and common practices, we could choose the entries in the restriction matrix, to expand on this model with Structural Vector Autoregression. We could dig deeper into a finer-grained data set, for example, using quarterly data provided by professional data companies, to have better accuracy in the model. We could also separate out the GDP growth rate for United States from the GDP growth rate for Developed countries to better examine the effects of the Effective Federal Funds rate on other Developed countries.

Overall, the Vector Autoregression is a simple but powerful tool to capture the relationship between multiple variables as they are moving with each other.))

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